Bethe ansatz for higher spin eight-vertex models

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Corrigendum

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Due to a production error 'sinh' and 'cosh' were incorrectly replaced by '$\sin h$' and '$\cos h$', respectively, throughout the article. IOP Publishing wish to apologize to Dr Takebe for this mistake. The following equations are the correct versions.

P 6686:

$$\rho(x) = \sum_{n\in\mathbb{Z}} \frac{\mathrm{e}^{2\pi\,\mathrm{i}nx}}{2\cosh 2\pi n\eta t}. \tag{2.2.4}$$

p 6687:

$$\begin{aligned}\frac{1}{N}\log\frac{\Lambda_1}{\Lambda_2} &= \log\frac{\theta_{11}(x+\ \mathrm{i}\eta t;\, \mathrm{i}t)}{\theta_{11}(x-\ \mathrm{i}\eta t\,:\, \mathrm{i}t)} + \frac{1}{N}\sum_{j=1}^{N/2}\log\frac{\theta_{11}(x-x_j-2\mathrm{i}\eta t;\, \mathrm{i}t)}{\theta_{11}(x-x_j+2\mathrm{i}\eta t;\, \mathrm{i}t)}\\ &\xrightarrow{N\to\infty} -\frac{3\pi\,\mathrm{i}}{2} - \pi\ \mathrm{i}x - \sum_{n=1}^{\infty}\frac{\mathrm{i}\sin 2\pi nx}{n\cosh 2\pi n\eta t}\end{aligned} \tag{2.2.6}$$

$$\begin{aligned}-\beta f(\lambda) &= (\mathrm{constant}) + \log\theta_{11}(\lambda+2\ell\eta;\,\tau) - 2\pi t(\lambda-\eta)(1-4\ell\eta)\\ &\quad - \sum_{n=1}^{\infty}\frac{\sinh\pi nt(1-4\ell\eta)\ \sinh 2\pi nt(\lambda-\eta)}{n\sinh\pi nt\ \cosh 2\pi n\eta t}.\end{aligned} \tag{2.2.8}$$

p6689:

$$\sigma(x) = -\frac{1}{4\ell} - \sum_{n=1}^{\infty}\frac{\sinh\pi nt\ \sinh 2\pi n\eta t}{\sinh\pi nt(1-4\ell\eta)\ \sinh 4\pi n\ell\eta t\cosh 2\pi n\eta t}\cos 2\pi nx \tag{2.3.8}$$

$$\omega_-(x) = -\frac{2\ell-1}{2\ell} - \sum_{n=1}^{\infty}\frac{2\sinh 2\pi n(2\ell-1)\eta t}{\sinh 4\pi n\ell\eta t}\cos 2\pi nx \tag{2.3.9}$$

$$\omega_+(x) = -1 - \sum_{n=1}^{\infty}\frac{2\sinh\pi nt(1-2(2\ell+1)\eta)}{\sinh\pi nt(1-4\ell\eta)}\cos 2\pi nx. \tag{2.3.10}$$

p6691:

$$J_n = \frac{\sinh 2\pi n(2\ell-1)\eta t}{2\pi\ \mathrm{i}n \sinh 4\pi n\ell\eta t}\left(\mathrm{e}^{-2\pi \mathrm{i}nx_-} - \frac{\mathrm{e}^{-2\pi\mathrm{i}nx_1}+\mathrm{e}^{-2\pi\mathrm{i}nx_2}}{2\cosh 2\pi n\eta t}\right) + \frac{\sinh \pi n(1-2(2\ell+1)\eta)t}{2\pi\ \mathrm{i}n \sinh \pi n(1-4\ell\eta)t}\left(\mathrm{e}^{-2\pi \mathrm{i}nx_+} - \frac{\mathrm{e}^{-2\pi\mathrm{i}nx_1}+\mathrm{e}^{-2\pi\mathrm{i}nx_2}}{2\cosh 2\pi n\eta t}\right). \qquad (2.3.21)$$

p6693:

$$\omega_0(x) = \sum_{n=1}^{\infty} \frac{2\sinh 2\pi n\eta t}{\sinh \pi n(1-4\ell\eta)t}\cos 2\pi nx. \qquad (2.3.33)$$

p6694:

$$J_n = \frac{\sinh 2\pi n(2\ell-1)\eta t}{\sinh 4\pi n\ell\eta t} \times\left(\mathrm{e}^{-2\pi\mathrm{i}nx_-} - \frac{-\mathrm{e}^{-\pi\ \mathrm{i}n}+\mathrm{e}^{-2\pi\mathrm{i}nx_1}+\mathrm{e}^{-2\pi\mathrm{i}nx_2}}{2\cosh 2\pi n\eta t} - \frac{\mathrm{e}^{-2\pi\ \mathrm{i}nx_0}}{\sinh \pi nt(1-4\ell\eta)}\right) + \frac{\sinh \pi n(1-2(2\ell+1)\eta)t}{2\cosh 2\pi n\eta t\sinh \pi n(1-4\ell\eta)t}(-\mathrm{e}^{-\pi\ \mathrm{i}n}+\mathrm{e}^{-2\pi\mathrm{i}nx_1}+\mathrm{e}^{-2\pi\mathrm{i}nx_2}). \qquad (2.3.43)$$

p6695:

$$\log\tau(x) := -\frac{\pi\mathrm{i}}{2} - \pi\ \mathrm{i}x - \mathrm{i}\sum_{n=1}^{\infty}\frac{\sin 2\pi nx}{n\cosh 2\pi n\eta t} \qquad (2.3.48)$$

(see [18] for details of calculations). This means that all conserved quantities such as momentum $P(x)$ or energy over the ground states are split into two terms:

$$P(x) = -\pi x - \sum_{n=1}^{\infty}\frac{\sin 2\pi nx}{n\cosh 2\pi n\eta t}$$

p6696:

The main contribution comes from the momentum P of the particle as $\mathrm{i}PN$, the *free phase*:

$$\mathrm{i}PN = -\mathrm{i}N\pi x - \mathrm{i}N\sum_{n=1}^{\infty}\frac{\sin 2\pi nx}{n\cosh 2\pi n\eta t}.$$

p6697:

$$\mathrm{i}\log(\pm S_{\mathrm{I}}(x)) = \sum_{n=1}^{\infty}\left(\frac{\sinh \pi nt(1-4\ell\eta-2\eta)}{n\sinh \pi nt(1-4\ell\eta)\ \cosh 2\pi n\eta t} + \frac{\sinh \pi nt(4\ell\eta-2\eta)}{n\sinh 4\pi n\ell\eta t\ \cosh 2\pi n\eta t}\right)\sin 2\pi nx + \sum_{n=1}^{\infty}\frac{2\sinh \pi nt(4\ell\eta-2\eta)}{n\sinh 4\pi n\ell\eta t}\sin \pi nx + \sum_{n=1}^{\infty}\frac{\sinh \pi nt(2\eta-(1-4\ell\eta))}{n\sinh \pi nt(1-4\ell\eta)}\sin \pi n(x-\varepsilon) \qquad (2.3.52)$$

$$\begin{aligned}
\mathrm{i}\log(\pm S_{\mathrm{II}}(x)) = \sum_{n=1}^{\infty} \Bigg(& \frac{\sinh \pi n t(1-4\ell\eta-2\eta)}{n \sinh \pi n t(1-4\ell\eta)\ \cosh 2\pi n \eta t} \\
& + \frac{\sinh \pi n t(4\ell\eta - 2\eta)}{n \sinh 4\pi n \ell \eta t\ \cosh 2\pi n \eta t} \Bigg) \sin 2\pi n x \\
& + \sum_{n=1}^{\infty} \frac{2 \sinh \pi n t(4\ell\eta-2\eta)}{n \sinh 4\pi n \ell \eta t} \sin \pi n x \\
& + \pi + \pi x + \sum_{n=1}^{\infty} \frac{\sinh 2\pi n \eta t}{n \sinh \pi n t(1-4\ell\eta)} \sin \pi n (x-\varepsilon)
\end{aligned} \tag{2.3.53}$$

p6698:

$$\begin{aligned}
\mathbb{S}(\mathrm{i}\lambda t;\mu) &= \exp\left(\sum_{n=1}^{\infty} \frac{\sinh \pi n t(\mu-2\eta)}{n \sinh \pi n t \mu\ \cosh 2\pi n t \eta} \sin 2\pi n\, \mathrm{i}\lambda t \right) \\
&= \frac{(q^4 p^{\lambda}; p^{\mu}, q^4)_\infty (p^{\lambda+\mu}; p^{\mu}, q^4)_\infty (q^2 p^{-\lambda}; p^{\mu}, q^4)_\infty (q^2 p^{-\lambda+\mu}; p^{\mu}, q^4)_\infty}{(q^4 p^{-\lambda}; p^{\mu}, q^4)_\infty (p^{-\lambda+\mu}; p^{\mu}, q^4)_\infty (q^2 p^{\lambda}; p^{\mu}, q^4)_\infty (q^2 p^{\lambda+\mu}; p^{\mu}, q^4)_\infty} \\
&= \frac{\Gamma_{q^4}\left(\frac{1}{2}+\frac{\lambda}{4\eta}\right)\Gamma_{q^4}\left(1-\frac{\lambda}{4\eta}\right)}{\Gamma_{q^4}\left(\frac{1}{2}-\frac{\lambda}{4\eta}\right)\Gamma_{q^4}\left(1+\frac{\lambda}{4\eta}\right)} \\
&\quad \times \prod_{k=1}^{\infty} \frac{\Gamma_{q^4}\left(\frac{1}{2}+\frac{\lambda+k\mu}{4\eta}\right)^2 \Gamma_{q^4}\left(1+\frac{-\lambda+k\mu}{4\eta}\right)\Gamma_{q^4}\left(\frac{-\lambda+k\mu}{4\eta}\right)}{\Gamma_{q^4}\left(\frac{1}{2}+\frac{-\lambda+k\mu}{4\eta}\right)^2 \Gamma_{q^4}\left(1+\frac{\lambda+k\mu}{4\eta}\right)\Gamma_{q^4}\left(\frac{\lambda+k\mu}{4\eta}\right)}
\end{aligned} \tag{2.3.56}$$

p6704:

$$\Phi(x;\mathrm{i}\mu t) = -2\pi x - 2\sum_{n=1}^{\infty} \frac{\sinh \pi n(1-2\mu)t}{n \sinh \pi n t} \sin 2\pi n x. \tag{C.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Phi(x;\mathrm{i}\mu t) = -2\pi\left(1 + 2\sum_{n=1}^{\infty} \frac{\sinh \pi n(1-2\mu)t}{\sinh \pi n t} \cos 2\pi n x\right). \tag{C.2}$$

$$\Psi(x;\mathrm{i}\mu t) = 2\sum_{n=1}^{\infty} \frac{\sinh 2\pi n \mu t}{n \sinh \pi n t} \sin 2\pi n x. \tag{C.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Psi(x;\mathrm{i}\mu t) = 4\pi \sum_{n=1}^{\infty} \frac{\sinh 2\pi n \mu t}{\sinh \pi n t} \cos 2\pi n x. \tag{C.4}$$

Lemma C.2. For $0 < a < b$, a series

$$\sum_{n\in\mathbb{Z}} \frac{\sinh \pi n a}{\sinh \pi n b} \mathrm{e}^{2\pi\,\mathrm{i} n x}$$

is positive for $x \in \mathbb{R}$. Here the term $n = 0$ is understood as a/b.

Proof. Define a function $f(y;x)$ by

$$f(y;x)=\frac{\sinh \pi ya}{\sinh \pi yb}e^{2\pi i yx}$$

$f(0;x)=a/b.$

p6705:
By Poisson's summation formula we have

$$\sum_{n\in\mathbb{Z}}\frac{\sinh \pi na}{\sinh \pi nb}e^{2\pi i nx}=\sum_{n\in\mathbb{Z}}f(n;x)=\sum_{n\in\mathbb{Z}}\hat{f}(n;x)>0.$$

Errors also occurred in *Proposition 1.2.2*. The correct version is as follows:
Proposition 1.2.2. Each component of $L_{k,k'}$ acts on the intertwining vector as follows.

$$\begin{aligned}
\alpha_{k,k'}(\lambda;s)\phi_{k,k'}&=W\left(\begin{matrix}k & k'\\ k-1 & k'-1\end{matrix}\middle|\lambda\right)\phi_{k-1,k'-1}\\
\beta_{k,k'}(\lambda;s)\phi_{k,k'}&=W\left(\begin{matrix}k & k'\\ k-1 & k'+1\end{matrix}\middle|\lambda\right)\phi_{k-1,k'+1}\\
\gamma_{k,k'}(\lambda;s)\phi_{k,k'}&=W\left(\begin{matrix}k & k'\\ k+1 & k'-1\end{matrix}\middle|\lambda\right)\phi_{k+1,k'-1}\\
\delta_{k,k'}(\lambda;s)\phi_{k,k'}&=W\left(\begin{matrix}k & k'\\ k+1 & k'+1\end{matrix}\middle|\lambda\right)\phi_{k+1,k'+1}
\end{aligned}\tag{1.2.5}$$

where $\phi_{k,k'}=\phi^{(\ell)}_{k,k'}(0;s)$ and W is the Boltzmann weight of SOS type [5, 9]:

$$\begin{aligned}
W\left(\begin{matrix}k & k'\\ k-1 & k'-1\end{matrix}\middle|\lambda\right)&=2\theta_{11}(\lambda+(k-k')\eta)\frac{\theta_{11}(w_{(k+k'+2\ell)/2})}{\theta_{11}(w_k)}\\
W\left(\begin{matrix}k & k'\\ k-1 & k'+1\end{matrix}\middle|\lambda\right)&=2\theta_{11}((k'-k-2\ell)\eta)\frac{\theta_{11}(w_{(k+k')/2}+\lambda)}{\theta_{11}(w_k)\theta_{11}(w_{k'})}\\
W\left(\begin{matrix}k & k'\\ k+1 & k'-1\end{matrix}\middle|\lambda\right)&=2\theta_{11}((k-k'-2\ell)\eta)\theta_{11}(w_{(k+k')/2}-\lambda)\\
W\left(\begin{matrix}k & k'\\ k+1 & k'+1\end{matrix}\middle|\lambda\right)&=2\theta_{11}(\lambda-(k-k')\eta)\frac{\theta_{11}(w_{(k+k'-2\ell)/2})}{\theta_{11}(w'_k)}.
\end{aligned}\tag{1.2.6}$$

This proposition is proved in the same way as (3.7) of [34].